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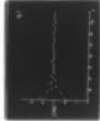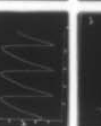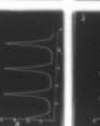   APR 79   M MANGEL
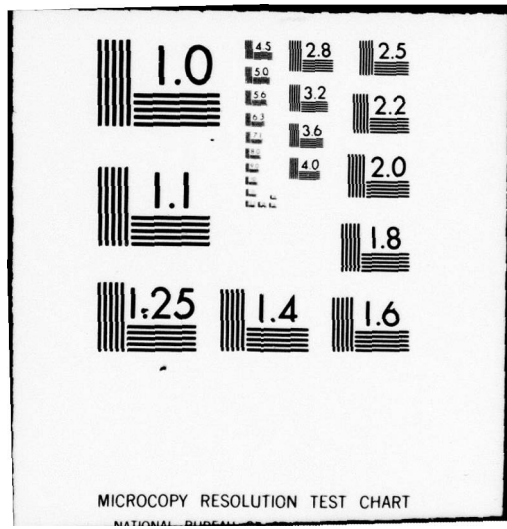UNCLASSIFIED   OEG-PROFESSIONAL PAPER-25   NL

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MODELING FLUCTUATIONS IN MACROSCOPIC SYSTEMS

Marc Mangel

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MODELING FLUCTUATIONS IN MACROSCOPIC SYSTEMS

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April 1979

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ABSTRACT

Macroscopic systems are often modeled by deterministic differential equations (DDE), such as $\dot{x}=f(x,t)$. Here, $x(t)$ is a macrovariable and represents an average over some set of ensembles. A possible extention of such a model to include fluctuations is to assume that a random variable $X(t)$ satisfies a stochastic differential equation (SDE), $\dot{X}=f(X,t)+a(X,t)\xi(t)$. In some sense, $x(t)$ should be the average of $X(t)$. If f is linear and $a(X,t)$ is a constant then $E\{X(t)\}=x(t)$. If $f(X,t)$ is non-linear, then $E\{f(X,t)\} \neq f(E(X),t)$ generally and some authors feel that the SDE is not a correct extension of the DDE. A procedure will be introduced here so that an appropriate conditional average of $X(t)$ is $x(t)$. Thus, there is an underlying consistency between the deterministic and stochastic formulations. The procedure also provides a prescription for the calculation of $a(X,t)$, which is usually not constant if $f(x,t)$ is non-linear. Two examples are studied to illustrate the application of the procedure. First, the logistic equation of population dynamics is studied in deterministic and stochastic versions. Second, stochastic effects on a chemical oscillator are analyzed.

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INTRODUCTION

Very often, macroscopic equations are modeled by differential equations of the form:

$$\frac{dx}{dt} = f(x,t) \qquad x(0) = x_0 .$$ (1)

In equation 1, $x = (x_1, x_2, \ldots, x_n)$ is a vector in R^n. Since the system of interest in macroscopic, $x(t)$ is regarded as an "ensemble average" of some sort. Namely, $x(t)$ is the average of some random variable $X(t)$. A possible way to find the equation that $X(t)$ satisfies is to add a random term to equation 1, so that $X(t)$ satisfies the Langevin equation:

$$\frac{dX}{dt} = f(X,t) + a(X,t)\xi(t) , \qquad X(0) = x_0 .$$ (2)

In equation 2, $\xi(t)$ is assumed to be a zero mean random process with correlation function $\phi(t,\tau) = E\{\xi(t+\tau)\xi(t)\}$. The connection between the two equations is not automatically clear and, in fact, is the subject of controversy [1,2]. In this paper, an attempt is made to relate the two equations. In particular, since experience shows that phenomenological equations are often deterministic (i.e., of the form (1)), "the mean" of $X(t)$ in equation 2 should be $x(t)$ in equation 1. The exact sense of "the mean" will be clarified in this paper.

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THE LINEAR CASE

The situation is simple (and classical, see references 3 and 4) when $f(x,t) = \Lambda x$, where Λ is a constant matrix. Assuming that $a(X,t) = A$, a constant, the stochastic equation 2 becomes:

$$\frac{dX}{dt} = \Lambda X + A\xi(t) \tag{3}$$

Averaging both sides in equation 3 gives:

$$\frac{d\langle X \rangle}{dt} = \Lambda\langle X \rangle \qquad \langle X(0) \rangle = x_0 , \tag{4}$$

where $\langle X \rangle = E\{X(t) | X(0) = x_0\}$. Clearly, in this case $\langle X(t) \rangle = x(t)$.

THE NON-LINEAR CASE

Now consider equation 2 when $f(\cdot,t)$ is non-linear. Averaging this equation gives:

$$\frac{d\langle X \rangle}{dt} = \langle f(X,t) \rangle + \langle a(X,t)\xi(t) \rangle . \tag{5}$$

When $f(\cdot,t)$ is non-linear

$$\langle f(X,t) \rangle \neq f(\langle X \rangle,t) , \tag{6}$$

except in special cases. Also note that in general,

$$\langle a(X,t)\xi(t) \rangle \neq 0 . \tag{7}$$

Thus, one concludes that in this case, $\langle X(t) \rangle \neq x(t)$ and that there is no simple relationship between equations 2 and 1.

Should equation 2 be rejected as meaningless? One could retrieve equation 1 from equation 2, by assuming that $a(X,t) \propto 1/V$ where V is a measure of the size of the system. Then, as $V \to \infty$, equation 2 collapses into equation 1. Such a procedure is used in statistical mechanics, of course, and is called the "thermodynamic limit" (with additional assumptions). In this paper, a different resolution of the paradox is given.

INITIALLY AND CONTINUOUSLY CONDITIONED MEANS*

The mean introduced in the previous section will be called the _initially conditioned mean_ since it is defined as:

$$\langle X(t) \rangle = E\{X(t) | X(0)=x_0\} \quad . \tag{8}$$

A second type of mean will be introduced now (also see reference 6). To motivate the results given below, note that equation 1 is equivalent to the difference equation:

$$x(t+\Delta t) - x(t) = f(x,t)\Delta t + o(\Delta t) \quad , \tag{9}$$

where $o(Z)/Z \to 0$ as $Z \to 0$. The interpretation of equation 9 is that _given_ $x(t) = x$, the increment in $x(t)$, $x(t+\Delta t) - x(t)$, is $f(x,t)\Delta t + o(\Delta t)$.

Denote by $\bar{X}(t)$ the _continuously conditioned mean._ Namely, define $\bar{X}(t)$ incrementally by:

$$\bar{X}(t+\Delta t) - \bar{X}(t) \equiv E\{X(t+\Delta t)-X(t) | X(t)=x\} \quad . \tag{10}$$

Now consider equation 2, given that $X(t) = x$. This equation can be rewritten as:

$$X(t+\Delta t) - X(t) = f(x,t)\Delta t + a(x,t)\xi(t)\Delta t + o(\Delta t) \quad . \tag{11}$$

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*These terms are due D.T. Gillespie, also see reference 5.

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Next, take the mean of equation 11, conditioned on $X(t) = x$.
Then, one finds:

$$\bar{X}(t+\Delta t) - \bar{X}(t) = f(x,t)\Delta t + o(\Delta t). \tag{12}$$

The right-hand sides of equations 2 and 9 are identical, so that
one concludes that:

$$\bar{X}(t) = x(t). \tag{13}$$

Thus, the deterministic variable $x(t)$ is the continuously
conditioned mean of $X(t)$. In order to exploit this concept, one
needs a fluctuation algorithm for the calculation of the continuously
conditioned mean. In the next section, such algorithms will be
discussed. These algorithms also lead to a prescription for the
calculation of $a(X,t)$. From equation 11, one has that, given
$X(t) = x$:

$$\left[X_i(t+\Delta t) - X_i(t)\right]\left[X_j(t+\Delta t) - X_j(t)\right] = \left[f_i(x,t)\Delta t + a_{ik}(x,t)\xi_k(t)\Delta t + o(\Delta t)\right]$$

$$\left[f_j(x,t)\Delta t + a_{jk}(x,t)\xi_k(t)\Delta t + o(\Delta t)\right]. \tag{14}$$

The notation used in equation 14 is that of summation over repeated
indices, so that:

$$a_{ik}(x,t)\xi_k(t) = \sum_{k=1}^{n} a_{ik}(x,t)\xi_k(t) \quad . \tag{15}$$

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If the noise process is to be noticable on deterministic time scales, one assumes that:

$$E\left[\xi_k(t)\xi_\ell(t)(\Delta t)^2\right] = \sigma_{k\ell}^2 \Delta t \quad.$$ (16)

Equation 16 is satisfied by Gaussian white noise and related approximating process (e.g., reference 7). Set:

$$\Delta X_i = X_i(t+\Delta t) - X_i(t) \quad.$$ (17)

Taking the continuously conditioned mean of equation 14 and using equations 15 and 16 gives:

$$\lim_{\Delta t \downarrow 0}\left[\frac{1}{\Delta t} E\{\Delta X_i \Delta X_j | X(t)=x\}\right] = \lim_{\Delta t \downarrow 0} a_{ik}(x,t)a_{j\ell}(x,t)$$

$$E\{\xi_k(t)\Delta t \xi_\ell(t)\Delta t\}$$ (18)

$$= a_{ik}(x,t)a_{j\ell}(x,t)\sigma_{k\ell}^2$$

Since σ_k^2 is presumed to be known, if the left-hand side of equation 18 is known, then the matrix $a_{ij}(x,t)$ can be determined.

Also note that in this formulation $a_{ij}(x,t)$ cannot be chosen arbitrarily but is defined by equation 18. Thus, by requiring the deterministic variable to be the continuously conditioned mean, the coefficient of the noise term is also fixed. Hence, it is not proper to assume in equation 2 that $a(x,t) = A$, a constant.

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PHENOMENOLOGICAL EQUATIONS
AND FLUCTUATION ALGORITHMS

In this section, two formalisms are given for the calculation of moments of ΔX_i. In the first algorithm, one assumes no knowledge of the mechanism underlying the macroscopic equations. In the second algorithm, one assumes that the underlying mechanism is known. Application of each algorithm is illustrated in the next section.

FLUCTUATION ALGORITHM FOR PROCESSES WITHOUT UNDERLYING MECHANISM

Often, the macroscopic equation is derived without knowledge of the underlying mechanism. The fluctuation algorithm is derived as follows. Assume that:

$$f_i(x,t) = f_i^+(x,t) - f_i^-(x,t) \quad , \tag{19}$$

with $f_i^+(x,t) \geq 0$, $f_i^-(x,t) \geq 0$. Then, assume that:

$$\Pr\{\Delta X_i = 1 \mid X(t) = x\} = f_i^+(x,t)\Delta t + o(\Delta t) \tag{20}$$

$$\Pr\{\Delta X_i = 1 \mid X(t) = x\} = f_i^-(x,t)\Delta t + o(\Delta t) \tag{21}$$

$$\Pr\{\text{all other transitions}\} = o(\Delta t) \quad . \tag{22}$$

Then, one finds:

$$E\{X_i(t+\Delta t) - X_i(t) \mid X(t) = x\} = \left[f_i^+(x,t) - f_i^-(x,t)\right]\Delta t + o(\Delta t) \quad , \tag{23}$$

and the matrix $[a_{ij}(x,t)]$ is diagonal with elements:

$$a_{ii}(x,t) = \left[\frac{f_i^+(x,t) + f_i^-(x,t)}{\sigma_{ii}^2}\right]^{\frac{1}{2}}. \tag{24}$$

Equation 24 is derived as follows. From equations 20-23,

$$E\{\Delta X_i \Delta X_j | X(t)=x\} = o(\Delta t) \quad , \tag{25}$$

if $i \neq j$. When $i = j$,

$$E\{\Delta X_i^2 | X(t)=x\} = [f_i^+(x,t) + f_i^-(x,t)]\Delta t + o(\Delta t). \tag{26}$$

Using equation 18 gives:

$$f_i^+(x,t) + f_i^-(x,t) = a_{ii}(x,t)^2 \sigma_{ii}^2 \quad , \tag{27}$$

from which equation 24 follows.

FLUCTUATION ALGORITHM FOR PROCESSES UNDERLYING MECHANISM

In many chemical and physical processes, one can use "elementary events" to describe the macroscopic events (see below for an example). Assume that there are M "elementary events" that can be modeled as:

$$Pr\{\Delta X_1 = r_1^k, \Delta X_2 = r_2^k, \ldots, \Delta X_n = r_n^k | X(t)=x\} = \lambda_k(x,t)\Delta t + o(\Delta t) \tag{28}$$

$$k = 1, 2, \ldots, M \ .$$

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From equation 28, one immediately obtains:

$$E\{\Delta X_i | X(t)=x\} = r_i^k \lambda_k(x,t)\Delta t + (\Delta t) . \tag{29}$$

For consistency of deterministic and stochastic equations, one requires that:

$$f_i(x,t) = r_i^k \lambda_k(x,t) . \tag{30}$$

From equation 28, one also immediately obtains:

$$E\{\Delta X_i \Delta X_j | X(t)=x\} = r_i^k r_j^k \lambda_k(x,t)\Delta t + o(\Delta t) . \tag{31}$$

Hence, the matrix $a_{ij}(x,t)$ is determined as the solution of:

$$r_i^k r_j^k \lambda_k(x,t) = a_{im}(x,t)a_{j\ell}(x,t)\sigma_{\ell m}^2 . \tag{32}$$

The fluctuation algorithm give here is equivalent to Keizer's "fluctuation dissipation postulates" (reference 1).

"SURE" TERMS IN THE MACROSCOPIC EQUATIONS

Sometimes a deterministic term will appear in the macroscopic equations. A typical case would be that $f(x,t)$ has a component of the form:

$$f_i(x,t) = f_{i0}(t) + f_{i1}(x,t) , \tag{33}$$

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where $f_{i0}(t)$ is deterministic. In this case, one rewrites equation 2 as:

$$\frac{dX_i}{dt} - f_{i0}(t) = f_{i1}(X,t) + a_{ik}(X,t)\xi_k(t) \quad . \tag{34}$$

From equation 34, it is apparent that the fluctuation algorithms described above should be applied to the quantity:

$$X_i(t+\Delta t) - X_i(t) - f_{i0}(t)\Delta t \quad , \tag{35}$$

(also, see reference 5).

EXAMPLES OF THE FLUCTUATION ALGORITHMS

In this section, two examples of the fluctuation algorithms are discussed. Two other examples are the mean field ferromagnet and the laser (references 8 and 9).

LOGISTIC EQUATION OF POPULATION DYNAMICS

The following equation is often used as a model in theoretical ecology (e.g., reference 10):

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) . \tag{36}$$

In this equation, $x(t)$ represents the size of the population at time t. R. May has suggested that the variable $X(t)$ satisfies the following stochastic equation (reference 10):

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) + \alpha X \xi(t) , \tag{37}$$

where $\xi(t)$ is Gaussian white noise and α is a constant.

If the first fluctuation algorithm is applied, then it is assumed that:

$$\text{Prob}\{X(t+\Delta t)-X(t)=1 | X(t)=x\} = rx\Delta t + o(\Delta t) \tag{38}$$

$$\text{Prob}\{X(t+\Delta t)-X(t)-1 | X(t)=x\} = \frac{rx^2}{K}\Delta t + o(\Delta t) \tag{39}$$

$$\text{Prob}\{\text{all other transitions}\} = o(\Delta t) . \tag{40}$$

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Equations 38-40 correspond to a linear birth-quadratic death process. Hence:

$$E\{(\Delta X)^2 | X(t)=x\} = rx\left(1 + \frac{x}{K}\right)\Delta t + o(\Delta t) \ . \tag{41}$$

For Gaussian white noise $\sigma^2 = 1$, so that equation 18 becomes:

$$rx\left(1 + \frac{x}{K}\right) = \left[a(x,t)\right]^2 \ . \tag{42}$$

Hence, the consistent extension of the logistic equation 36 is not equation 37 but is:

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) + \left[rX\left(1 + \frac{X}{K}\right)\right]^{\frac{1}{2}}\xi(t) \ . \tag{43}$$

The Fokker-Planck equation for the probability density of the process described by equation 37 is (see reference 7):

$$\frac{\partial}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}(aX^2\rho) - \frac{\partial}{\partial x}\left[rX\rho\left(1 - \frac{X}{K}\right)\right] \tag{44a}$$

While the Fokker-Planck equation for the density fo the process described by equation 43 is:

$$\frac{\partial \rho}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[rX\rho\left(1 + \frac{X}{K}\right)\right] - \frac{\partial}{\partial X}\left[rX\rho\left(1 - \frac{X}{K}\right)\right] \ . \tag{44b}$$

Both of the above equations are singular diffusion equations (reference 11). There is, however, a major difference in that the diffusion coefficient of equation 44a goes as X^2 for small X, but that of equation 44b goes as X for small X. Many of the

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results described in reference 10 are due to the high singularity of the diffusion coefficient. Some of these results would not be obtained if equation 43 were used instead of equation 37.

CHEMICAL OSCILLATOR WITH FLUCTUATIONS

The following reaction mechanism has been studied by I. Prigogine and his collaborators (reference 12) as a prototype chemical oscillator.

$$A \rightarrow X_1$$

$$X_1 + B \rightarrow X_2$$

$$2X_1 + X_2 \rightarrow 3X_1 \tag{45}$$

$$X_1 \rightarrow P$$
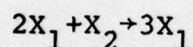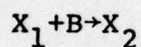
Assume that the concentrations of A and B, denoted by α, β respectively, are constant. Let x_i denote the concentration of species i. The phenomenological rate equations are:

$$\frac{dx_1}{dt} = \alpha + x_1^2 x_2 - (\beta + 1) x_1$$

$$\tag{46}$$

$$\frac{dx_2}{dt} = \beta x_1 - x_1^2 x_2$$

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These equations can have oscillatory solutions (figures 1 and 2). Hence, one concludes that if the stochastic problem is properly formulated, then the continuously conditioned means $\overline{X_1(t)}$, $\overline{X_2(t)}$ are also oscillatory.

Now consider the formulation of the stochastic equations. The equations for $X_1(t)$ and $X_2(t)$ take the form (assuming that α is a sure term):

$$\frac{dX_1}{dt} - \alpha = X_1^2 X_2 - (\beta+1)X_1 + \left[a_{11}(X_1,X_2,t)\xi_1(t) + a_{12}(X_1,X_2,t)\xi_2(t)\right] \tag{47}$$

$$\frac{dX_2}{dt} = \beta X_1 - X_1^2 X_2 + \left[a_{12}(X_1,X_2,t)\xi_2(t) + a_{22}(X_1,X_2,t)\xi_3(t)\right]. \tag{48}$$

Here, $\xi_1(t)$, $\xi_2(t)$, $\xi_3(t)$ are independent stochastic processes. They need not be Gaussian white noise.

The second fluctuation algorithm of the previous section applies in this case. From the reaction mechanism (equation 45) the following transition probabilities are constructed:

$$\Pr\{(\Delta X_1 - \alpha\Delta t)=1, \ \Delta X_2=-1 \mid X_1(t)=x_1, \ X_2(t)=x_2\} = x_1^2 x_2 \Delta t + o(\Delta t) \tag{49}$$

$$\Pr\{(\Delta X_1 - \alpha\Delta t)=-1, \ \Delta X_2=0 \mid X_1(t)=x_1, \ X_2(t)=x_2\} = x_1 \Delta t + o(\Delta t) \tag{50}$$

$$\Pr\{(\Delta X_1 - \alpha\Delta t)=-1, \ \Delta X_2=1 \mid X_1(t)=x_1, \ X_2(t)=x_2\} = \beta x_1 \Delta t + o(\Delta t) \tag{51}$$

$$\Pr\{\text{all other transitions}\} = o(\Delta t) \tag{52}$$

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From equations 49-52, one finds that:

$$E\{(\Delta X_1 - \alpha \Delta t)|X_1(t)=x_1,\ X_2(t)=x_2\} = x_1^2 x_2 \Delta t - (\beta+1)x_1 \Delta t + o(\Delta t) \qquad (53)$$

$$E\{\Delta X_2|X_1(t)=x_1,\ X_2(t)=x_2\} = \beta x_1 \Delta t - x_1^2 x_2 \Delta t + o(\Delta t)\ , \qquad (54)$$

So that the deterministic equations are obtained as the conditional mean.

Now consider calculation of the matrix (a_{ij}). From equations 49-52:

$$E\{(\Delta X_1 - \alpha \Delta t)^2|X_1(t)=x_1,\ X_2(t)=x_2\} = \left[x_1^2 x_2 + (\beta+1)x_1\right]\Delta t + o(\Delta t) \qquad (55)$$

Since $E\{\Delta X_1|X(t)=x\} = 0(\Delta t)$, equation 55 is rewritten as:

$$E\{(\Delta X_1)^2|X_1(t)=x_1,\ X_2(t)=x_2\}+o(\Delta t) = \left[x_1^2 x_2 + (\beta+1)x_1\right]\Delta t + o(\Delta t) \qquad (56)$$

Clearly,

$$E\{(\Delta X_2)^2|X_1(t)=x_1,\ X_2(t)=x_2\} = \left[x_1^2 x_2 + \beta x_1\right]\Delta t + o(\Delta t)\ . \qquad (57)$$

Finally,

$$E\{(\Delta X_1 - \alpha \Delta t)\Delta X_2|X_1(t)=x_1,\ X_2(t)=x_2\} = -(x_1^2 x_2 + \beta x_1)\Delta t + o(\Delta t). \qquad (58)$$

Expanding the product in the left-hand side of equation 58 gives

$$E\{\Delta X_1 \Delta X_2|X_1(t)=x_1,\ X_2(t)=x_2\}+o(\Delta t) = -(x_1^2 x_2 + \beta x_1)\Delta t + o(\Delta t). \qquad (59)$$

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The matrix (a_{ij}) is now determined from equation 18.

Since the continuously conditioned mean oscillates, it is interesting to wonder if the initially conditioned mean also oscillates (also see references 5 and 13). In figures 3 and 4, the initially conditioned means $\langle X_1(t) \rangle$ and $\langle X_2(t) \rangle$ are plotted. These means were evaluated by numerical solution of equations 47 and 48. It is seen that these menas decay and seem to approach a constant. The technique of reference 13 can be used to check the numerical results as follows. If the initially conditioned mean of equations 47 and 48 is taken, one obtains:

$$\frac{d}{dt}\langle X_1 \rangle = \langle X_1^2 X_2 \rangle + \alpha - (\beta+1)\langle X_1 \rangle$$
$$+ \langle a_{11}(X_1,X_2,t)\xi_1(t) + a_{12}(X_1,X_2,t)\xi_2(t) \rangle \tag{59}$$

$$\frac{d}{dt}\langle X_2 \rangle = \beta\langle X_1 \rangle - \langle X_1^2 X_2 \rangle$$
$$+ \langle a_{12}(X_1,X_2,t)\xi_2(t) + a_{22}(X_1,X_2,t)\xi_3(t) \rangle . \tag{60}$$

Now <u>assume</u> that as $t \to \infty$

$$\frac{d}{dt}\langle X_i \rangle \to 0 \qquad i = 1,2 \tag{61}$$

and

$$\langle a_{ik}(X_1,X_2,t)\xi_k(t) \rangle \to 0 \qquad i,k = 1,2 . \tag{62}$$

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Equations 61 and 62 are assumptions and are not derived from any of the above results. Denote by $\langle X_i \rangle_\infty$ as the limiting value of $\langle X_i(t) \rangle$ as $t \to \infty$. Then, from equations 59 and 60:

$$0 = \langle X_1^2 X_2 \rangle_\infty + \alpha - (\beta+1)\langle X_1 \rangle_\infty \qquad (63)$$

$$0 = \beta \langle X_1 \rangle_\infty - \langle X_1^2 X_2 \rangle_\infty \quad . \qquad (64)$$

Hence, one finds,

$$\langle X_1 \rangle_\infty = \alpha \quad , \qquad (65)$$

which appears to be in agreement with the numerical result in figure 3. Equations 63 and 64 cannot be used to provide information about $\langle X_2 \rangle_\infty$.

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REFERENCES

1. Keizer, J., J. Chem. Phys. 63:5037 (1975).

2. Van Kampen,N.G., Adv. Chem. Phys. 34:245 (1976).

3. Onsager, L. and S. Machlup, Phys. Rev. 91:1505 (1953), 91:1512 (1953).

4. Groot, S.R. and P. Mazur, <u>Non-Equilibrium Thermodynamics,</u> (North Holland, Amsterdam, 1962).

5. Gillespie, D.T., M. Mangel, and J.W. Turner, Manuscript in Preparation.

6. Mangel, M., J. Chem. Phys. 69:3697 (1978).

7. Wong, E., Stochastic Processes in Information and Dynamical Systems, McGraw Hill, New York,1971.

8. a) Kubo, R., K. Matsuo, and K. Kitahara, J. Stat. Phys. 9:51 (1973).

8. b) Mangel,M., Physica A, Accepted.

9. Risken, H., Prog. in Optics, 8:241 (1970.

10. May, R., Stability and Complexity in Model Ecosystems, (Princeton University Press, Princeton, N.J., 1973).

11. Feller, W., Annals of Math., 55:468 (1952).

12. Nicolis, G. and I. Prigogine, Self-Organization in Non-Equilibrium Systems, Wiley Interscience, New York, 1977.

13. Feistal, R. and W. Ebeling, Physica 93A:114 (1978).

Acknowledgements

CAPTIONS:

Fig. 1: The continuously conditioned mean $\overline{X_1(t)}$ for the chemical oscillator (45), with $\alpha=1$, $\beta=3$.

Fig. 2: The continuously conditioned mean $\overline{X_2(t)}$.

Fig. 3: The initially conditioned mean $\langle X_1(t) \rangle$ for the chemical oscillator (45). An ensemble of 500 phase points was used.

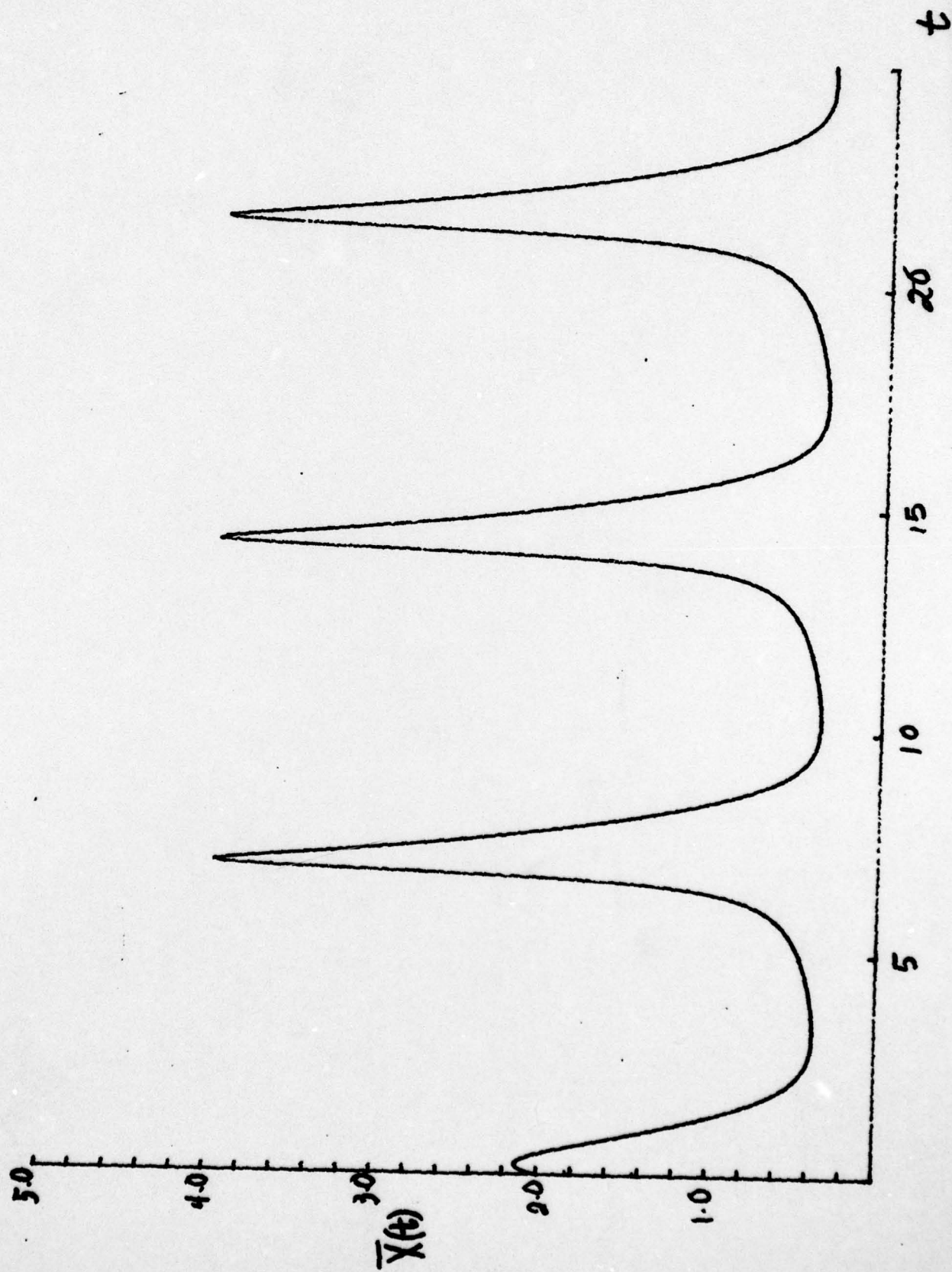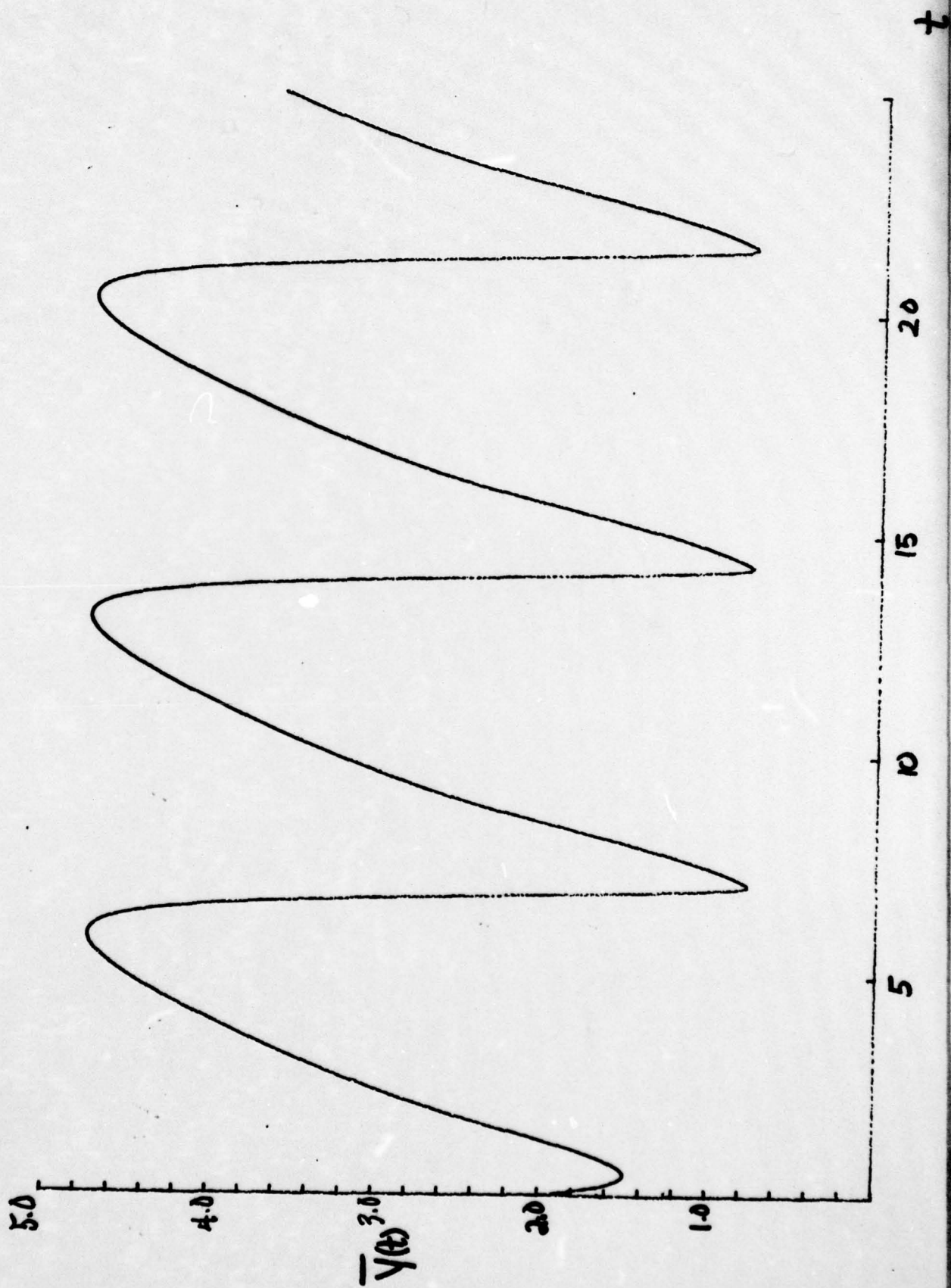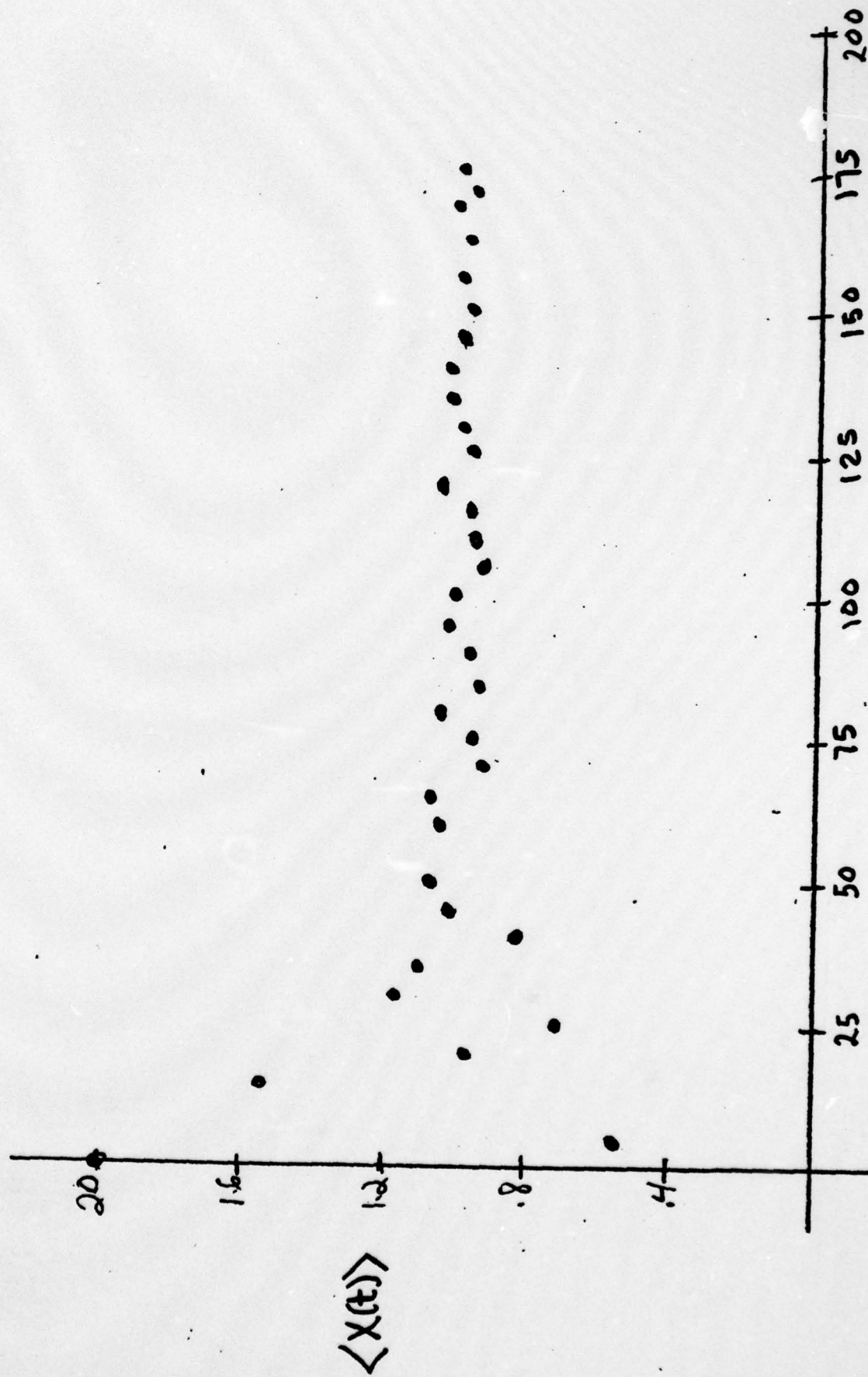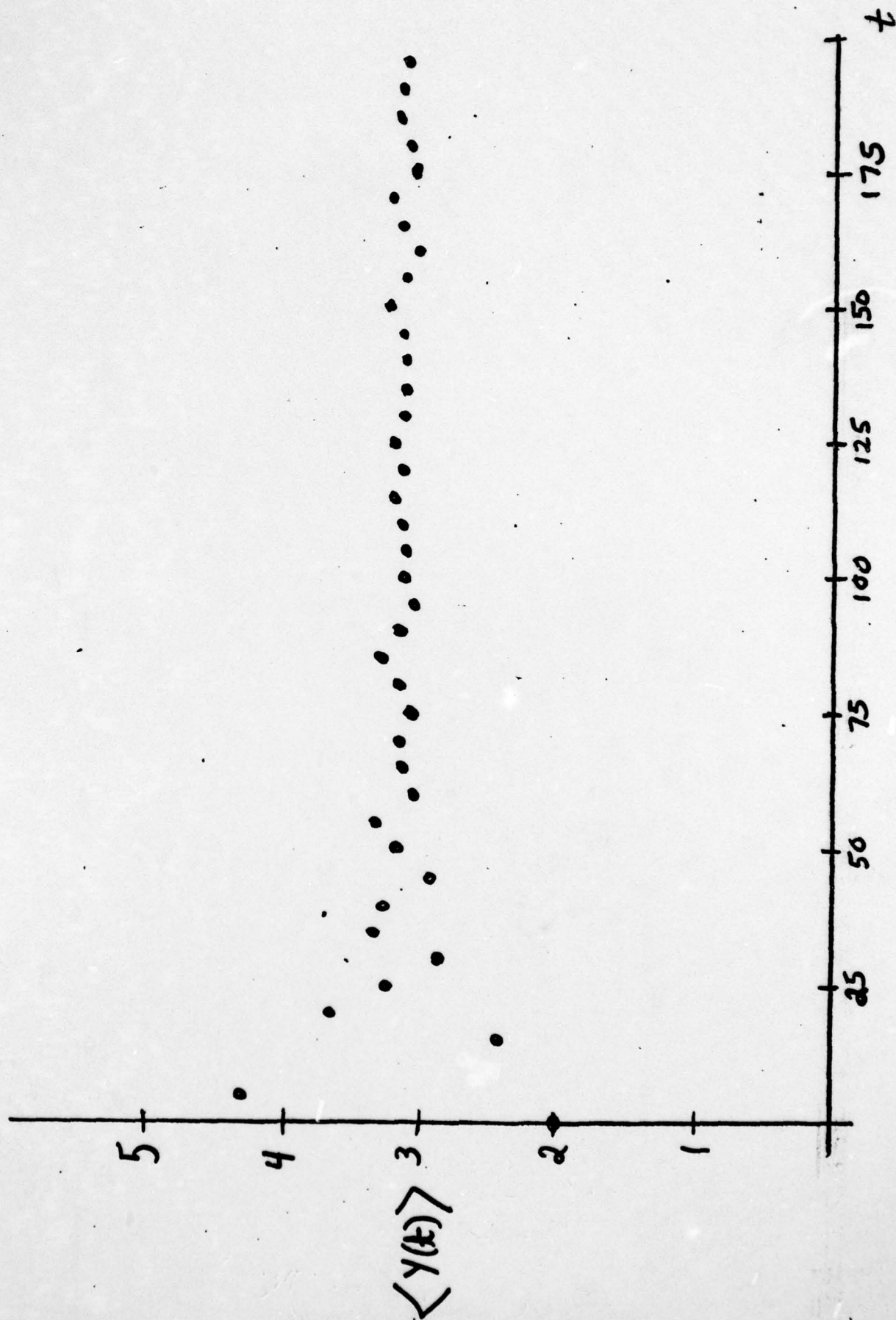Fig. 4: The initially conditioned mean $\langle X_2(t) \rangle$.

Fig 1

Fig 2

Fig. 3

Fig. 4

CNA Professional Papers — 1973 to Present*

PP 103
Friedheim, Robert L., "Political Aspects of Ocean Ecology" 48 pp., Feb 1973, published in Who Protects the Oceans, John Lawrence Hargrove (ed.) (St. Paul: West Publ'g. Co., 1974), published by the American Society of International Law) AD 757 936

PP 104
Schick, Jack M., "A Review of James Cable, Gunboat Diplomacy Political Applications of Limited Naval Forces," 5 pp., Feb 1973, (Reviewed in the American Political Science Review, Vol. LXVI, Dec 1972)

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Corn, Robert J. and Phillips, Gary R., "On Optimal Correction of Gunfire Errors," 22 pp., Mar 1973, AD 761 674

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Stoloff, Peter H., "User's Guide for Generalized Factor Analysis Program (FACTAN)," 35 pp., Feb 1973, (Includes an addendum published Aug 1974) AD 758 824

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Stoloff, Peter H., "Relating Factor Analytically Derived Measures to Exogenous Variables," 17 pp., Mar 1973, AD 758 820

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Augusta, Joseph, "A Critique of Cost Analysis," 9 pp., Jul 1973, AD 766 376

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McWhite, Peter B. and Ratliff, H. Donald,* "Defending a Logistics System Under Mining Attack,"** 24 pp., Aug 1976 (to be submitted for publication in Naval Research Logistics Quarterly), presented at 44th National Meeting, Operations Research Society of America, November 1973, AD A030 454
*University of Florida.
**Research supported in part under Office of Naval Research Contract N00014-68-0273-0017

PP 118
Barfoot, C. Bernard, "Markov Duels," 18 pp., Apr 1973, (Reprinted from Operations Research, Vol. 22, No. 2, Mar-Apr 1974)

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Stoloff, Peter and Lockman, Robert F., "Development of Navy Human Relations Questionnaire," 2 pp., May 1974, (Published in American Psychological Association Proceedings, 81st Annual Convention, 1973) AD 779 240

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Smith, Michael W. and Schrimper, Ronald A.,* "Economic Analysis of the Intracity Dispersion of Criminal Activity," 30 pp., Jun 1974, (Presented at the Econometric Society Meetings, 30 Dec 1973) AD 780 538
*Economics, North Carolina State University.

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Devine, Eugene J., "Procurement and Retention of Navy Physicians," 21 pp., Jun 1974, (Presented at the 49th Annual Conference, Western Economic Association, Las Vegas, Nev., 10 Jun 1974) AD 780 539

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Kelly, Anne M., "The Soviet Naval Presence During the Iraq-Kuwaiti Border Dispute: March-April 1973," 34 pp., Jun 1974, (Published in Soviet Naval Policy, ed. Michael MccGwire; New York: Praeger) AD 780 592

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Petersen, Charles C., "The Soviet Port-Clearing Operation in Bangladesh, March 1972-December 1973," 35 pp., Jun 1974, (Published in Michael MccGwire, et al. (eds) Soviet Naval Policy: Objectives and Constraints, (New York: Praeger Publishers, 1974) AD 780 540

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Friedheim, Robert L. and Jehn, Mary E., "Anticipating Soviet Behavior at the Third U.N. Law of the Sea Conference: USSR Positions and Dilemmas," 37 pp., 10 Apr 1974, (Published in Soviet Naval Policy, ed. Michael MccGwire; New York: Praeger) AD 783 701

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Weinland, Robert G., "Soviet Naval Operations—Ten Years of Change," 17 pp., Aug 1974, (Published in Soviet Naval Policy, ed. Michael MccGwire; New York: Praeger) AD 783 962

PP 126 — Classified.

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Stoloff, Peter and Lockman, Robert F., "Evaluation of Naval Officer Performance," 11 pp., (Presented at the 82nd Annual Convention of the American Psychological Association, 1974) Aug 1974, AD 784 012

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Holen, Arlene and Horowitz, Stanley, "Partial Unemployment Insurance Benefits and the Extent of Partial Unemployment," 4 pp., Aug 1974, (Published in the Journal of Human Resources, Vol. IX, No. 3, Summer 1974) AD 784 010

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Weinland, Robert G., "Analysis of Gorshkov's Navies in War and Peace," 45 pp., Aug 1974, (Published in Soviet Naval Policy, ed. Michael MccGwire; New York: Praeger) AD 786 319

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*Department of Mechanical Engineering, University of Maryland.

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Goudreau, Kenneth A.; Kuzmack, Richard A.; Wiedemann, Karen, "Analysis of Closure Alternatives for Naval Stations and Naval Air Stations," 47 pp., 3 Jun 1975 (Reprinted from "Hearing before the Subcommittee on Military Construction of the Committee on Armed Service," U.S. Senate, 93rd Congress, 1st Session, Part 2, 22 Jun 1973)

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*Mathematica, Inc.

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*Research supported by the National Science Foundation

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Lockman, Robert F., Jehn, Christopher, and Shughart, William F. II, "Models for Estimating Premature Losses and Recruiting District Performance," 36 pp., Dec 1975 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A 020 443

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Horowitz, Stanley and Sherman, Allan (LCdr., USN), "Maintenance Personnel Effectiveness in the Navy," 33 pp., Jan 1976 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A021 581

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Horowitz, Stanley A., "The Economic Consequences of Political Philosophy," 8 pp., Apr 1976 (Reprinted from Economic Inquiry, Vol. XIV, No. 1, Mar 1976)

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Mizrahi, Maurice M., "On Path Integral Solutions of the Schrodinger Equation, Without Limiting Procedure,"* 10 pp., Apr 1976 (Reprinted from Journal of Mathematical Physics, Vol. 17, No. 4 (Apr 1976), 566-575).
*Research supported by the National Science Foundation

PP 153

Mizrahi, Maurice M., "WKB Expansions by Path Integrals, With Applications to the Anharmonic Oscillator,"* 137 pp., May 1976, AD A025 440
*Research supported by the National Science Foundation

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Mizrahi, Maurice M., "On the Semi-Classical Expansion in Quantum Mechanics for Arbitrary Hamiltonians," 19 pp., May 1976 (Published in Journal of Mathematical Physics, Vol. 18, No. 4, p. 786, Apr 1977), AD A025 441

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Stoloff, Peter H. and Balut, Stephen J., "Vacate: A Model for Personnel Inventory Planning Under Changing Management Policy," 14 pp. April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 049

PP 180
Horowitz, Stanley A. and Sherman, Allan, "The Characteristics of Naval Personnel and Personnel Performance," 16 pp. April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 050

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Balut, Stephen J. and Stoloff, Peter, "An Inventory Planning Model for Navy Enlisted Personnel," 35 pp., May 1977, (Prepared for presentation at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science. 9 May 1977, San Francisco, California), AD A042 221

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Murray, Russell, 2nd, "The Quest for the Perfect Study or My First 1138 Days at CNA," 57 pp., April 1977

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PP 184
Lockman, Robert F., "An Overview of the OSD/ONR Conference on First Term Enlisted Attrition," 22 pp., June 1977, (Presented to the 39th MORS Working Group on Manpower and Personnel Planning, Annapolis, Md., 28-30 June 1977), AD A043 618

PP 185
Kassing, David, "New Technology and Naval Forces in the South Atlantic," 22 pp. (This paper was the basis for a presentation made at the Institute for Foreign Policy Analyses, Cambridge, Mass., 28 April 1977), AD A043 619

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Mizrahi, Maurice M., "Phase Space Integrals, Without Limiting Procedure," 31 pp., May 1977, (Invited paper presented at the 1977 NATO Institute on Path Integrals and Their Application in Quantum Statistical, and Solid State Physics, Antwerp, Belgium, July 17-30, 1977) (Published in Journal of Mathematical Physics 19(1), p. 298, Jan 1978), AD A040 107

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Coile, Russell C., "Nomography for Operations Research," 35 pp., April 1977, (Presented at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Services, San Francisco, California, 9 May 1977), AD A043 620

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Coile, Russell C., "Error Detection in Computerized Information Retrieval Data Bases," July, 1977, 13 pp. Presented at the Sixth Cranfield International Conference on Mechanized Information Storage and Retrieval Systems, Cranfield Institute of Technology, Cranfield, Bedford, England, 26-29 July 1977, AD A043 580

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Weinland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1976 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture"), Chicago, Ill., September 2, 1976)

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Coile, Russell C., "Comments on: Principles of Information Retrieval by Manfred Kochen, 10 pp., Mar 78, (Published as a Letter to the Editor, Journal of Documentation, Vol. 31, No. 4, pages 298-301, December 1975)

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Coile, Russell C., "Lotka's Frequency Distribution of Scientific Productivity," 18 pp., Feb 1978, (Published in the Journal of the American Society for Information Science, Vol. 28, No. 6, pp. 366-370, November 1977)

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Mangel, Marc, "Aggregation, Bifurcation, and Extinction In Exploited Animal Populations*," 48 pp., Mar 1978 (To be submitted for publication in American Naturalist)
*Portions of this work were started at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, B.C., Canada

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